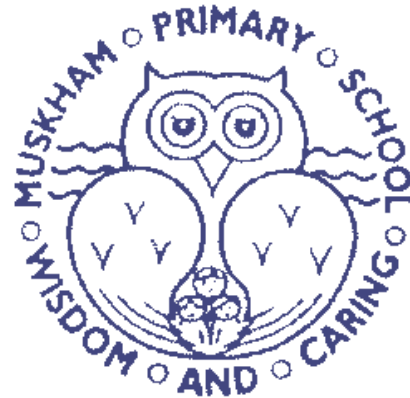


Muskham Primary school



Use of Manipulatives and  
Pictorial Representations in the Teaching of Maths

Spring 2023

## Introduction

At Muskham Primary school, we use the concrete -pictorial- abstract approach in the teaching of Maths. This policy outlines the concrete objects and images used in teaching the four operations: Addition, Subtraction, Multiplication and Division. It is to be used in conjunction with the Calculation Policy.

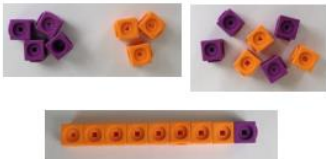


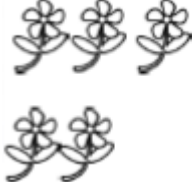
### Manipulatives

Concrete is the “doing” stage. All the practical apparatus that we use in our classrooms such as Multilink cubes, Dienes apparatus, counters, place value counters, bead strings and also those that use numerals such as place value cards, hundred squares, digit cards, dice and dominoes are all practical bits of equipment that children can pick up and manipulate. They have intrinsically various aspects of numbers and the number system in them to help children to get to grips with the very abstract notions of numbers, the relationships between them and the ways in which they work in the number system. Manipulatives can be powerful tools to support sense making, mathematical thinking and reasoning when they are used as tools to support these processes rather than as adjuncts to blindly following a taught procedure to arrive at an answer.

### Pictorial Representations

Pictorial is the “seeing” stage. This helps children to make connections between the concrete and abstract stage; the abstract stage uses mathematical symbols to represent problems. A range of representations (pictures) are used to secure children’s understanding before moving to the abstract stage.

# Addition

	Manipulatives	Pictorial Representations
<b>Foundation onwards</b>	<p>Use of objects e.g. cubes to show combining of 2 sets of objects</p>  <p>Use of Numicon This shows <math>2 + 1 = 3</math></p>  <p>Using fingers: <math>5 + 1</math></p> 	<p>Pictures of objects to count the total:</p>  <p>This shows <math>3 + 2 = 5</math></p>

## Y1 onwards

### Bead strings

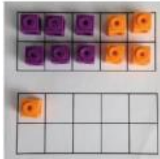


Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

### Ten Frames

#### Regrouping to make 10



$$6 + 5 = 11$$



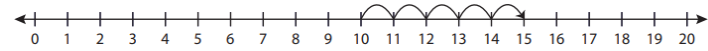
Start with the bigger number and use the smaller number to make 10.

#### 6 + 5 – Regrouping to make 10



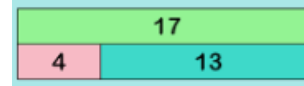
Place a  on it and  to show 11

### Use of number lines:



Start at the bigger number and count on

### Bar models:



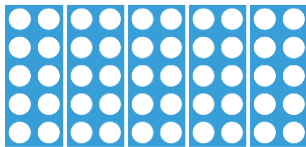
Using a square to represent 10 and a circle to represent 1

$$13 + 5 = \square \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$$

## Y2 onwards

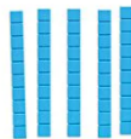
### Adding multiples of 10 using Numicon:

$$30 + 20$$

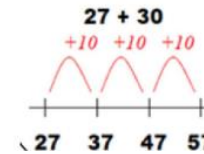


### Adding multiples of 10 using Base 10:

$$30 + 20$$

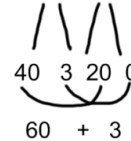


### Using a number line:



### Using looping:

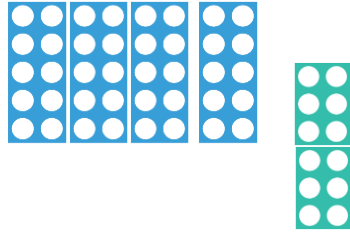
$$43 + 20 = 63$$



Adding two 2-digit numbers by exchanging:

Using Numicon

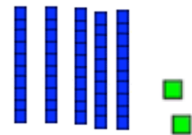
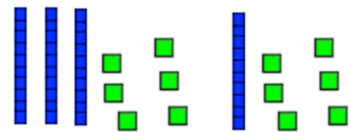
$$36 + 16$$



Place a '10' Numicon on top of the two '6' Numicons  $50 + 2 = 52$

Using Base 10

$$36 + 16$$



Adding three 1-digit numbers

Using Numicon

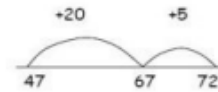
$$5 + 6 + 3$$



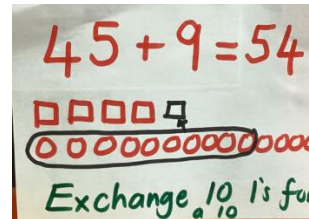
This is equal to  $10 + 4 = 14$

Adding a pair of 2-digit numbers using a condensed partition method:

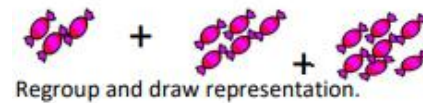
$$43 + 25 = 68$$



$$47 + 25 \text{ Bridging 10}$$



Show exchanging using squares and circles



$$= 15$$

Using Cubes:



Combine to make 10 first if possible and then add the third digit

### Year 3 onwards

Column Addition: Adding 3 digit to a 2digit using place value counters (Column Addition) with no regrouping:

Hundreds	Tens	Ones

$$533 + 22$$

Column Addition: Regrouping

--	--

$$46 + 27 = 73$$

Represent the place value counters by drawing into a grid

2 3 5 + 4 5 8 =

H	T	O
•••	•••	•••
•••	•••	•••
		•••
		•••
		•••

## Year 4 onwards

Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

Hundreds	Tens	Ones

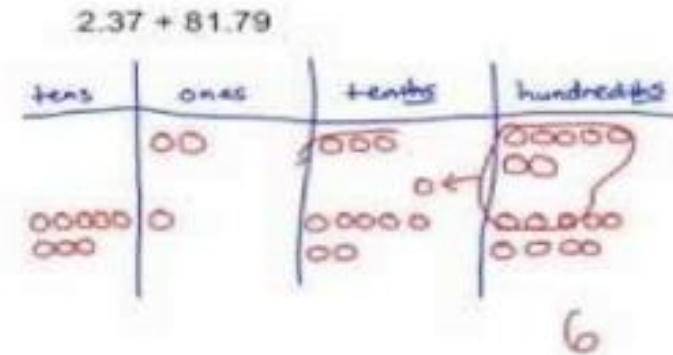
7	1	5	1

Draw representations using place value grid.

## Year 5 onwards

Adding numbers with 4 digits or more  
Introduce decimal place value counters

1,000s	100s	10s	1s



# Subtraction

## Foundation onwards

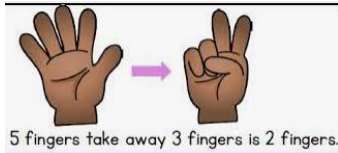
Use physical objects, counters, cubes etc to show how objects can be taken away.



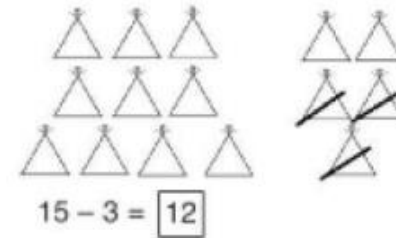
Use Numicon



Use fingers:



Cross out drawn objects to show what has been taken away.



# Year 1 onwards

Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

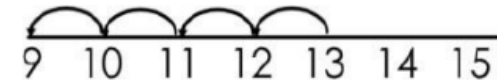


Use counters and move them away from the group as you take them away counting backwards as you go.

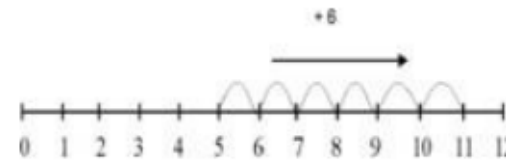


Use cubes to build towers or make bars to find the difference

Count back on a number line or number track



$$13 - 4 = 9$$

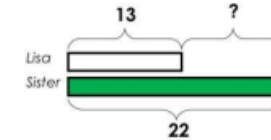


Count on to find the difference.

### Comparison Bar Models

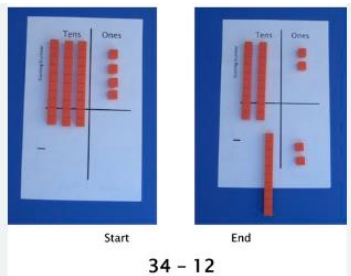
Draw bars to find the difference between 2 numbers.

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



# Y2 onwards

Use Base 10:

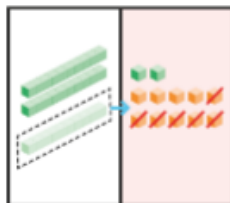


$$34 - 12$$

Crossing 10:

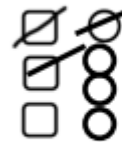
$$32 - 16$$

Regroup one of the tens into 10 ones.



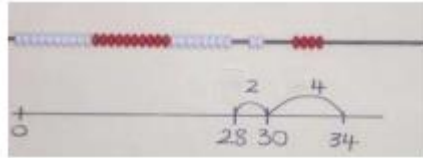
Draw using squares and circles:

$$34 - 21 =$$



$$52 - 23 =$$

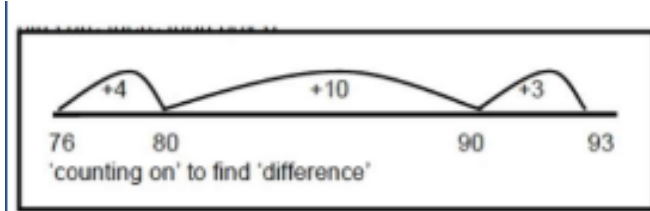




$$34 - 28$$

Use a bead bar or bead strings to model counting to next ten and the rest.

Counting on:  $93 - 76$

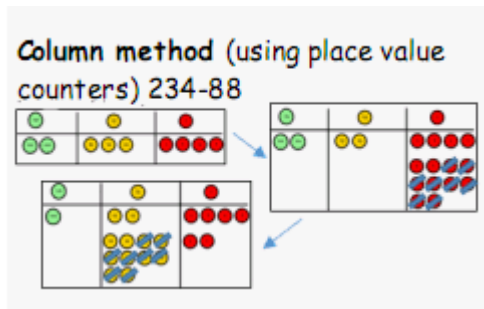


Use a number line to count on to next multiple of ten; then count on in multiples of 10 and finally count on the remaining number of ones. Add the total number of jumps together.

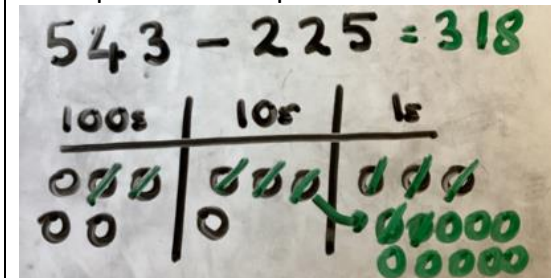
## Y3 onwards

Introducing partitioning column method:  
With and without regrouping - revisit using Dienes and place value counters.

Move to place value counters, modelling the exchange of a ten into 10 ones and hundred into 10 tens.

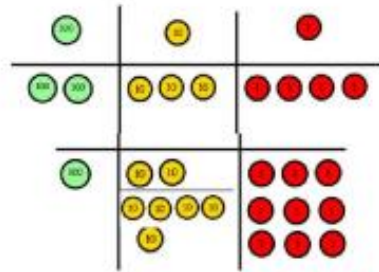


Draw place value representations



**Years 4 – 6**  
**Increasingly larger numbers**

$$234 - 179$$



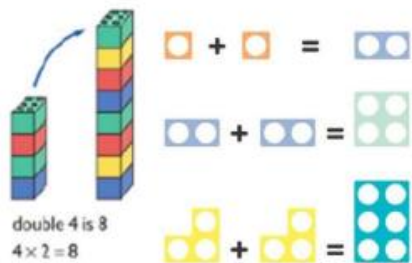
Model process of exchange using Numicon, base ten and then move to PV counters.

As for Year 3

# Multiplication

## Foundation onwards

Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling



Draw pictures to show how to double numbers:

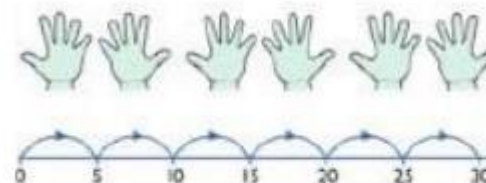
Double 4 is 8



Double 3 is 6

## Year 1 onwards

Counting in multiples



Use a number line or pictures to continue support in counting in multiples.

Counting in 5s



Making equal groups:



3 equal groups of 2

Repeated addition



$$5 + 5 + 5 + 5$$

Arrays

Use Numicon:



2 lots of 3

Build arrays with counters:



repeated addition

Draw pictures to represent equal groups and

Draw pictures of arrays



## Year 2 onwards

Multiplication is commutative

Use Numicon:



2 lots of 4



4 lots of 2

Arrays:



2 X 3 is equal to 3 x 2

Hands:

3 x 2



count in 2s on each finger 2,4,6

For 3 x 10, count in 10s on each finger 10,20,30

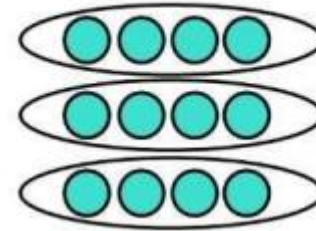
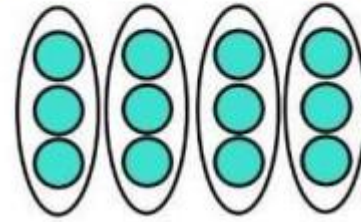
For 3 x 5, count in 5s on each finger 5,10,15

Beadstrings:



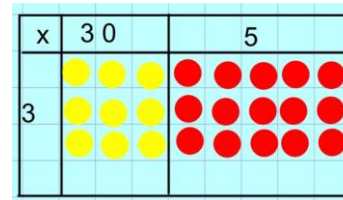
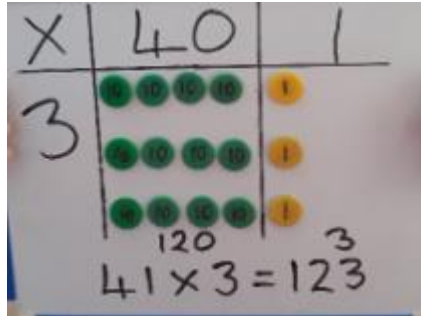
$3 \times 5 = 5 + 5 + 5$

Use representations of arrays to show different calculations and explore commutativity.



### Year 3 onwards

Multiplying a 2-digit number by a single digit number  
Use place value counters to introduce the grid method

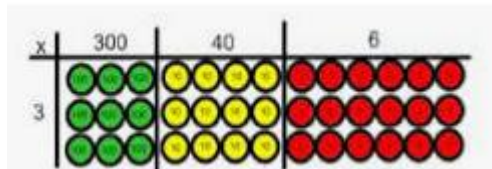


Children draw the arrays into the grid.

X	20	3
8	160	24

### Year 4 onwards

Extend to multiplying a 3-digit number by a single digit using place value counters



x	300	20	7
4	1200	80	28

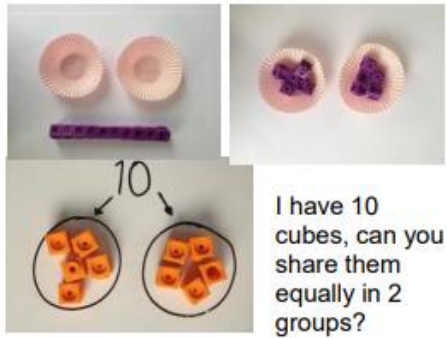
### Year 5 onwards

Extend to multiplying a 4-digit number by a 1- or 2-digit number using place value counters

# Division

## Foundation onwards

Sharing objects equally:



Share 8 between 2

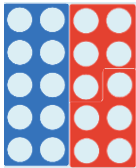
Children use pictures or shapes to share quantities equally.



## Year 1 onwards

Sharing:

10 divided by 2 is 5 using Numicon



As well as sharing introduce grouping.

Divide quantities into equal groups. Use Numicon, cubes, counters, objects or place value counters to aid understanding.

I have 10 biscuits, I give 2 to each child, how many children can get biscuits? **How many groups of 2 in 10?**

How many groups of 2 in 10?



Bar models for sharing:

12 shared between 2

12	



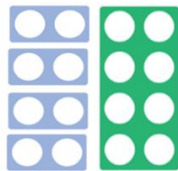
I have 12 chairs. I put 4 chairs around each table, how many tables do I need?



**How many groups of 4 can I make with 12?**

Use Numicon:

How many groups of 2 in 8?



Use a bead string:

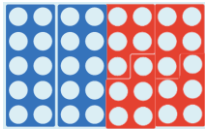
How many groups of 5 in 20?



## Year 2 onwards

Using Numicon:

$$20 \div 5$$



Place four 5s on 20 to show the answer 4

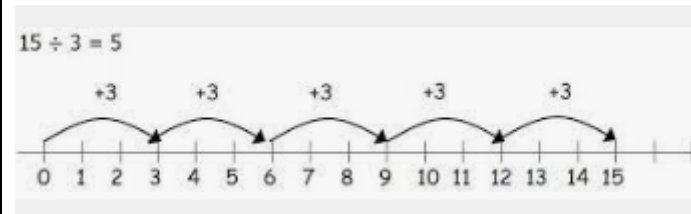
Make arrays:

How many groups of 3 in 6?

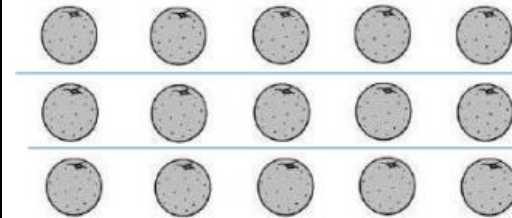
How many groups of 2 in 6?



Number line:



How many groups of 5 in 30?



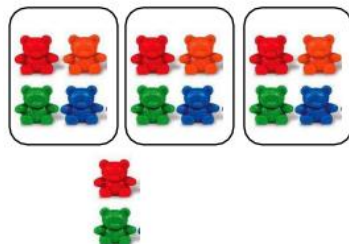
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

## Y3 onwards

Introducing remainders when dividing

$$14 \div 3 =$$

Divide objects between groups and see how much is left over



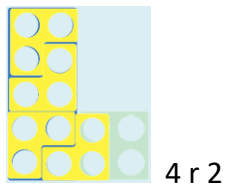
Draw dots and group them to divide an amount and clearly show a remainder.



$$14 \div 3 = 4 \text{ r } 2$$

Using Numcion:

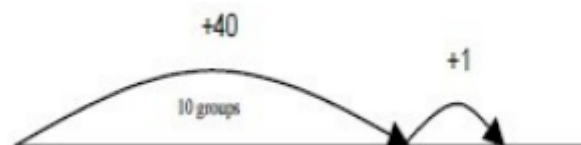
$14 \div 3$



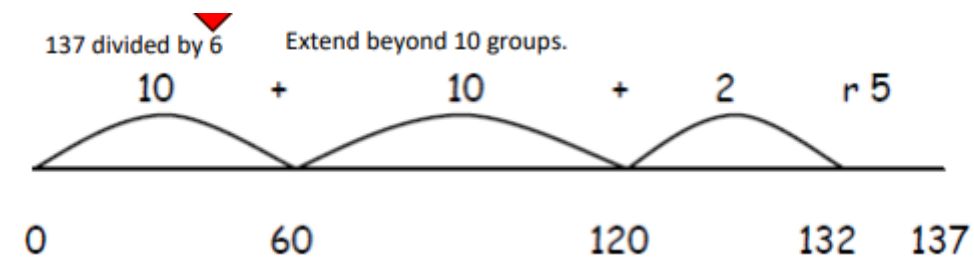
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



$13 \div 4 = 3 \text{ r } 1$



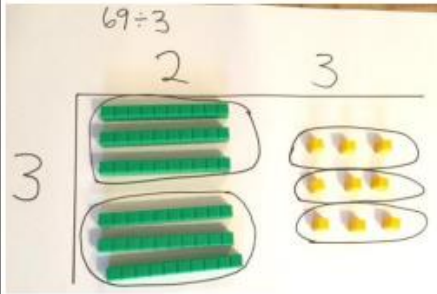
$41 \div 4 = 10 \text{ r } 1$



## Y4 onwards

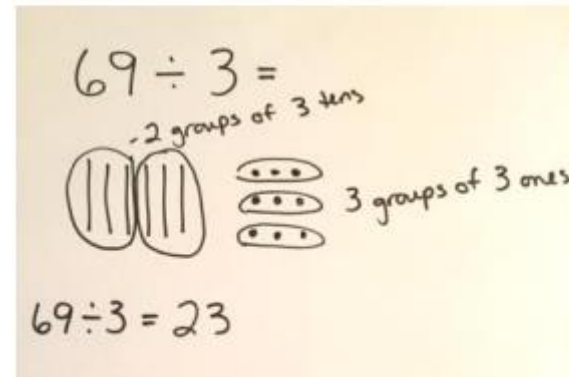
### Short Division:

When dividing 2 digit numbers children begin by representing the number with Dienes. They then see how many groups of the divisor they can make:



I can make 2 groups of 3 tens. I can make 3 groups of 3 ones.

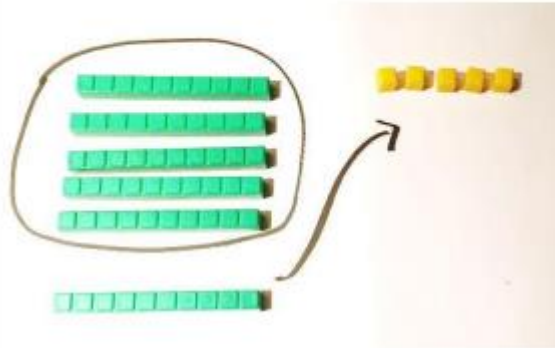
Children can use drawings to represent the Dienes (or they can draw place value counters) and see how they can be grouped:



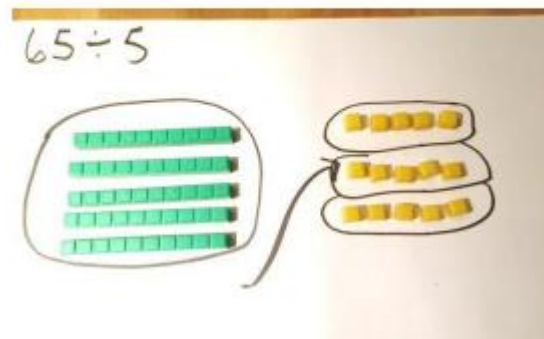
With regrouping:

$$65 + 5 =$$

Step 1:

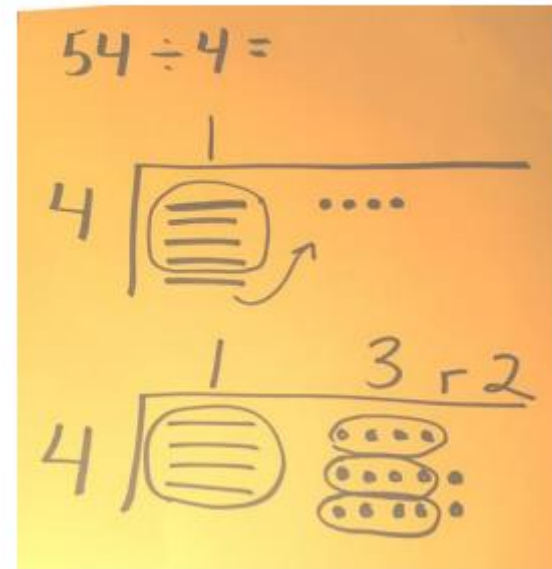


Step 2:



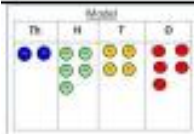
Just a thought: Do we need to include fingers too?

$$54 \div 4 = 13 \text{ r } 2$$



# Y6 onwards

## Long division



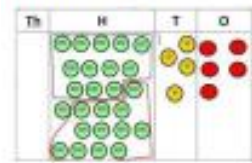
2544 ÷ 12  
 How many groups of 12 thousands do we have?  
 None

Exchange 2 thousand for 20 hundreds.



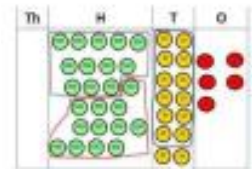
$$12 \overline{) 2544} \begin{array}{r} 0 \\ \hline \end{array}$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.  
 We have grouped 24 hundreds so can take them off and we are left with one.



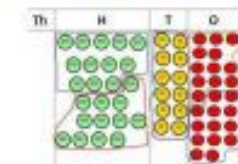
$$12 \overline{) 2544} \begin{array}{r} 02 \\ \hline 24 \\ \hline 1 \\ \hline \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12 \overline{) 2544} \begin{array}{r} 021 \\ \hline 24 \\ \hline 14 \\ \hline 12 \\ \hline 2 \\ \hline \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12 \overline{) 2544} \begin{array}{r} 0212 \\ \hline 24 \\ \hline 14 \\ \hline 12 \\ \hline 24 \\ \hline 24 \\ \hline 0 \\ \hline \end{array}$$

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